Large-Margin Thresholded Ensembles for Ordinal Regression

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Reduction Method

Algorithmic

1. identify the type of learning problem (ordinal regression)
2. find premade reduction (thresholded ensemble) and oracle learning algorithm (AdaBoost)
3. build a ordinal regression rule using (ORBoost) + data

Theoretical

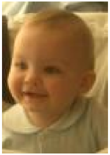
1. identify the type of learning problem (ordinal regression)
2. find premade reduction (thresholded ensemble) and known generalization bounds (large-margin ensembles)
3. derive new bound (large-margin thresholded ensembles) using the reduction + known bound

this work: a concrete instance of reductions

Ordinal Regression

- what is the age-group of the person in the picture?

2



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- rank: a finite ordered set of labels $\mathcal{Y} = \{1, 2, \cdots, K\}$
- ordinal regression:
  given training set $\{(x_n, y_n)\}_{n=1}^{N}$, find a decision function g that predicts the ranks of unseen examples well
- e.g. ranking movies, ranking by document relevance, etc.

matching human preferences:
applications in social science and info. retrieval

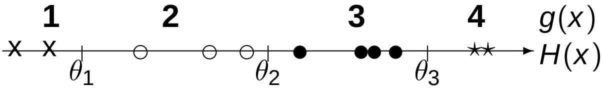
Properties of Ordinal Regression

- regression without metric:
  - possibly metric underlying (age), but not encoded in $\{1, 2, 3, 4\}$
- classification with ordered categories:
  - small mistake – classify a teenager as a child; big mistake – classify an infant as an adult
- common loss functions:
  - determine the category: classification error $L_C(g, x, y) = \big[g(x) \neq y \big]$
  - or at least have a close prediction: absolute error $L_A(g, x, y) = \big| g(x) - y \big|$

will talk about L_A only;
similar for L_C

Thresholded Model for Ordinal Regression

- naive algorithm for ordinal regression:
  1. do general regression on $\{(x_n, y_n)\}$, and get $H(x)$
     – general regression performs badly without metric
  2. set $g(x) = \text{clip}(\text{round}(H(x)))$
     – roundoff operation (uniform quantization) cause large error
- improved and generalized algorithm:
  1. estimate a potential function $H(x)$
  2. quantize $H(x)$ by some ordered θ to get $g(x)$



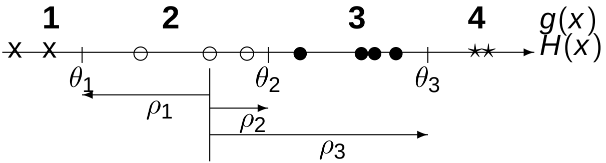
thresholded model: $g(x) \equiv g_{H,\theta}(x) = \min\{k : H(x) < \theta_k\}$

Thresholded Ensemble Model

- the potential function $H(x)$ is a weighted ensemble $H(x) \equiv H_T(x) = \sum_{t=1}^{T} w_t h_t(x)$
- intuition: combine preferences to estimate the overall confidence
- e.g. if many people, h_t, say a movie x is "good", the confidence of the movie $H(x)$ should be high
- h_t can be binary, multi-valued, or continuous
- $w_t < 0$: allow reversing bad preferences

thresholded ensemble model:
ensemble learning for ordinal regression

Margins of Thresholded Ensembles



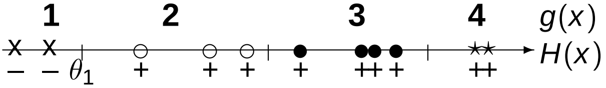
- margin: safe from the boundary
- normalized margin for thresholded ensemble

$$\bar{\rho}(x, y, k) = \left\{ \begin{array}{l} H_T(x) - \theta_k, \text{ if } y > k \\ \theta_k - H_T(x), \text{ if } y \leq k \end{array} \right\} \Bigg/ \left(\sum_{t=1}^{T} |w_t| + \sum_{k=1}^{K-1} |\theta_k| \right)$$

negative margin $\iff$ **wrong prediction**

$$\sum_{k=1}^{K-1} \left[\bar{\rho}(x, y, k) \leq 0 \right] \iff \left| g(x) - y \right|$$

Theoretical Reduction



- $(K-1)$ binary classification problems w.r.t. each θ_k: $\bigl((X)_k, (Y)_k\bigr) = \bigl((x,k), +/-\bigr)$
- (Schapire et al., 1998) binary classification: with probability at least $1-\delta$, for all $\Delta > 0$ and binary classifiers g_c,

$$\mathcal{E}_{(X,Y)\sim\mathcal{D}'}\left[g_c(X) \neq Y\right] \leq \frac{1}{N}\sum_{n=1}^{N}\left[\bar{\rho}(X_n, Y_n) \leq \Delta\right] + O\Bigl(\frac{\log N}{\sqrt{N}}, \frac{1}{\Delta}, \sqrt{\log \frac{1}{\delta}}\Bigr)$$

- (Lin and Li, 2006) ordinal regression: with similar settings, for all thresholded ensembles g,

$$\mathcal{E}_{(x,y)\sim\mathcal{D}} L_A(g,x,y) \leq \frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{K-1}\left[\bar{\rho}(x_n, y_n, k) \leq \Delta\right] + O\Bigl(K, \frac{\log N}{\sqrt{N}}, \frac{1}{\Delta}, \sqrt{\log \frac{1}{\delta}}\Bigr)$$

large-margin thresholded ensembles can generalize

Algorithmic Reduction

- (Freund and Schapire, 1996) AdaBoost: binary classification by operationally optimizing

$$\min \sum_{n=1}^{N} \exp\big(-\rho(x_n, y_n)\big) \approx \max \text{softmin}_n \; \bar{\rho}(x_n, y_n)$$

- (Lin and Li, 2006)

ORBoost-LR (left-right):

$$\min \sum_{n=1}^{N} \sum_{k=y_n-1}^{y_n} \exp\big(-\rho(x_n, y_n, k)\big)$$

ORBoost-All:

$$\min \sum_{n=1}^{N} \sum_{k=1}^{K-1} \exp\big(-\rho(x_n, y_n, k)\big)$$

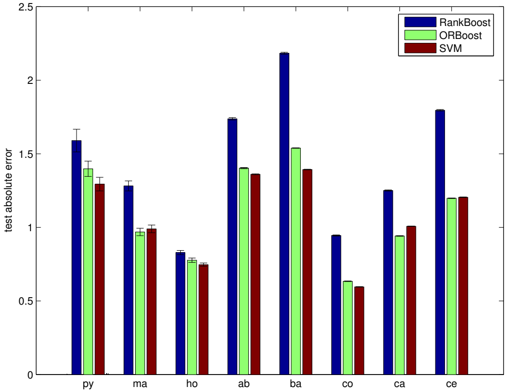
algorithmic reduction to AdaBoost

Advantages of ORBoost

- ensemble learning: combine simple preferences to approximate complex targets
- threshold: adaptively estimated scales to perform ordinal regression
- inherit from AdaBoost:
  - simple implementation
  - guarantee on minimizing $\sum_{n,k} \left[\bar{\rho}(x_n, y_n, k) \leq \Delta\right]$ fast
  - practically less vulnerable to overfitting

useful properties inherited with reduction

ORBoost Experiments



Results (ORBoost-All)

- ORBoost-All simpler, and much better than RankBoost (Freund et al., 2003)
- ORBoost-All much faster, and comparable to SVM (Chu and Keerthi, 2005)
- similar for ORBoost-LR

Conclusion

- thresholded ensemble model: useful for ordinal regression
  - theoretical reduction: new large-margin bounds
  - algorithmic reduction: new training algorithms – ORBoost
- ORBoost:
  - simplicity over existing boosting algorithms
  - comparable performance to state-of-the-art algorithms
  - fast training and less vulnerable to overfitting
- on-going work: similar reduction technique for other theoretical and algorithmic results with more general loss functions (Li and Lin, 2006)

Questions?